Appendix

(Note: The Proximity constraints referred to in the results get stated in the course of the proofs)

Lemma
The atomic Brier score satisfies Proximity 1

Proof of Lemma
Let *b* and *c* be credence functions defined over a finite set of worlds Ω, where the distance between worlds in Ω is given by the disagreement metric. Let w_a be any world in Ω and suppose that the multiset {*b*(w) | w ∈ Ω} can be mapped one-to-one onto the multiset {*c*(w) | w ∈ Ω}by the function *F* as follows:

i. If *b*(w) = *c*(w) then *F*(*b*(w)) = *c*(w)
ii. If *b*(w) ≠ *c*(w) then for some world w*, *F*(*b*(w)) = *c*(w*) and *F*(*b*(w*)) = *c*(w) and the following conditions are satisfied:
    a. The distance between w* and w_a differs from the distance between w and w_a
    b. *b* and *c*'s credences are swapped between w and w*, with *b* investing the larger credence in the closer world (to w_a) and the smaller credence in the further world.
    c. The further of the two worlds (w and w*) from w_a disagrees with w_a about all the atomic propositions that the closer of the two worlds disagrees with w_a about, in addition to disagreeing with w_a about at least one other atomic proposition (hence making it further).

We'll show that on the weighted Brier score which assigns equal weight to all the atomic propositions and no weight to any other propositions, *b* is at least as accurate as *c* at w_a, and if (ii) holds for at least one w ∈ Ω, *b* is more accurate than *c* at w_a.

Let the falsehoods concerning the atomic propositions at w_a be $\{P_1 \ldots P_m\}$. (In other words, if the atomic propositions are $\{A_1 \ldots A_m\}$, then if $w_a \in A_i$, $P_i = {\sim}A_i$, and if $w_a \notin A_i$ then $P_i = A_i$). The inaccuracy of *b* at w_a on the weighted Brier score is just the sum of the inaccuracy of *b* with respect to the P_i.[1]

Now, for any such proposition P_i, consider those worlds w in P_i such that *b* invests a larger credence in w than *c* does. Call these worlds $w_{i1} \ldots w_{im}$ (the i is just a reminder that we're listing worlds that are members of <u>P_i</u>).

So we have that for j∈ {1…m}, $b(w_{ij}) > c(w_{ij})$.

We know that for any such w_{ij}, there exists a partner world, which we'll call, w_{ij}* such that w_{ij}* is further from w than w_{ij} is, and which is such that $b(w_{ij}) = c(w_{ij}*)$, and $c(w_{ij}) = b(w_{ij}*)$. Now note that w_{ij}* is also a member of P_i. Why? Because w_{ij}* is the *further* of the two partners from w_a, and we've stipulated that the further world disagrees with w_a about all the atomic propositions that the closer world disagrees with w_a about. Since $w_{ij} \in P_i$ but $w_a \notin P_i$, w_{ij}* must be a member of P_i as well.

[1] Note that I'm relying here to the fact that the Brier score is symmetric.

Now we'll order the worlds that are members of P_i as follows: first will come the pairs of worlds, w_{ij}, $w_{ij}*$, where $b(w_{ij}) > c(w_{ij})$, and then will come all the remaining worlds which will be such that $b(w_{ij}) \leq c(w_{ij})$

So:

$I_{l\text{-}brier}(b(P_i), w_a(P_i)) = [b(w_{i1})+b(w_{i1}*)+\ldots b(w_{im})+b(w_{im}*)+b(w_{i(m+1)})+\ldots b(w_{i(m+n)})]^2$

For $j \in \{1\ldots m\}$ we can swap $b(w_{ij})$ with $c(w_{ij}*)$ and $b(w_{ij}*)$ with $c(w_{ij})$. So b's inaccuracy with respect to P_i:

$$= [c(w_{i1}*)+c(w_{i1})+\ldots c(w_{im}*)+c(w_{im})+b(w_{i(m+1)})+\ldots b(w_{i(m+n)})]^2$$

And recalling that, by construction of our ordering, for all $j \in \{m+1\ldots m+n\}$ $b(w_{ij}) \leq c(w_{ij})$, we have that the inaccuracy of $b(P_i)$ at w_a is

$$\leq [c(w_{i1}*)+c(w_{i1})+\ldots c(w_{im}*)+ c(w_{im})+c(w_{i(m+1)})+\ldots c(w_{i(m+n)})]^2 = I(c(P_i), w_a(P_i)).$$

So, for all P_i, the inaccuracy of b is less than or equal to the inaccuracy of c on the weighted Brier.

Suppose now that for some world $w \in \Omega$ condition (ii) obtains and $b(w) \neq c(w)$. Then there exists a partner world for w, w*, such that b and c swap credences between these worlds with b investing the larger credence in the closer world. Without loss of generality, suppose w* is the further world. Then w* disagrees with w_a about all the atomic propositions that w disagrees with w_a about in addition to at least one other atomic proposition. So let P_z be a falsehood concerning an atomic proposition such that $w* \in P_z$ but $w \notin P_z$.

Let's now think about b's inaccuracy with respect to P_z. As before we can express b's P_z-inaccuracy as:

$I_{l\text{-}Brier}(b(P_z), w_a(P_z) = [b(w_{z1})+b(w_{z1}*)+\ldots b(w_{zm})+b(w_{zm}*)+b(w_{z(m+1)})+\ldots b(w_{z(m+n)})]^2$

Since $w* \in P_z$

$w* \in \{w_{z1}, w_{z1}*\ldots w_{zm}, w_{zm}*, w_{z(m+1)}\ldots w_{z(m+n)}\}$.

Note that $w* \notin \{w_{z1}, w_{z2}\ldots w_{zm}\}$. This is because, for all w_{zj} where $j \in \{1\ldots m\}$, $b(w_{zj}) > c(w_{zj})$. However, since w* is the further world of {w, w*}, and b invests the smaller credence in the further world $b(w*)<c(w*)$.

Note also that $w* \notin \{w_{z1}*, w_{z2}*\ldots w_{zm}*\}$. For the $w_{zj}*$ are all partners of the w_{zj} worlds. This means that if $w* = w_{zj}*$ for some $j \in \{1\ldots m\}$, then w*'s partner would be w_{zj} for some $j \in \{1\ldots m\}$. But w*'s partner is w, and since w (by assumption) is not a member of P_z, w can't equal any such w_{zj}.

It follows that $w* \in \{w_{z(m+1)}\ldots w_{z(m+n)}\}$.

Since we know that $b(w^*) < c(w^*)$ (w^* is the further world) it follows that for some $w_{zj} \in \{w_{z(m+1)}\ldots w_{z(m+n)}\}$, $b(w_{zj}) < c(w_{zj})$.

Since for all $j\in \{m+1\ldots m+n\}$, $b(w_{zj}) \leq c(w_{zj})$ and for some $j\in \{m+1\ldots m+n\}$, $b(w_{zj}) < c(w_{zj})$ it follows that:

$$b(w_{z(m+1)})+b(w_{z(m+2)})+\ldots b(w_{z(m+n)}) < c(w_{z(m+1)})+c(w_{z(m+2)})+\ldots c(w_{z(m+n)})$$

Returning to b's inaccuracy with respect to P_z, we have:

$$I(b(P_z), w_a(P_z) = [b(w_{z1})+b(w_{z1}^*)+\ldots b(w_{zm})+b(w_{zm}^*)+b(w_{z(m+1)})+\ldots b(w_{z(m+n)})]^2$$

$$=[c(w_{z1}^*)+c(w_{z1})+\ldots c(w_{zm}^*)+c(w_{zm})+ b(w_{z(m+1)})+\ldots b(w_{z(m+n)})]^2$$

$$< [c(w_{z1}^*)+c(w_{z1})+\ldots c(w_{zm}^*)+c(w_{zm})+c(w_{z(m+1)})+\ldots c(w_{z(m+n)})]^2= I(c(P_z), w_a(P_z)).$$

Since for all P_i, b's inaccuracy with respect to P_i is less than or equal to c's, but for some P_i, b's inaccuracy is less than c's, it follows that b is less inaccurate than c at w_a on the weighted-Brier.

Result 1

Every global atomic inaccuracy measure derived from a local inaccuracy measure that satisfies TRUTH DIRECTEDNESS and SYMMETRY satisfies Proximity 1.

Proof of Result 1

Let g be a strictly increasing function representing the local inaccuracy of a credence in a falsehood and let this local inaccuracy measure satisfy SYMMETRY.[2] Then, in the proof of the lemma above, simply substitute any expression of the form $[\ldots]^2$ with $g[\ldots]$.

Result 2

Every global inaccuracy measure which assigns equal weight to all the at-most propositions and no other propositions, all the at-least propositions and no other propositions or both the at-most and at-least propositions and no other propositions, and which is derived from a local inaccuracy measure that satisfies TRUTH DIRECTEDNESS and SYMMETRY satisfies Proximity 2.

Proof of Result 2

Let b and c be credence functions defined over a finite set of worlds Ω, where the distance between worlds in Ω is given by the magnitude metric. Let w_a be any world in Ω and suppose that the multiset $\{b(w_i)\,|\,w_i \in \Omega\}$ can be mapped one-to-one onto the multiset $\{c(w_i)\,|\,w_i \in \Omega\}$by the function F as follows:

i. If $b(w_i) = c(w_i)$ then $F(b(w_i)) = c(w_i)$
ii. $b(w_i) \neq c(w_i)$ and there is some world w_j, such that the following conditions are satisfied:
    a. The distance between w_j and w_a differs from the distance between w_i and w_a.

[2] Note 19 explains the role that SYMMETRY plays in the proof.

b. *b* and *c*'s credences are swapped between the two worlds (w_i and w_j), with *b* investing the *larger* credence in the *closer* world (to w_a) and the *smaller* credence in the *further* world (from w_a).
c. i and j are both greater than a, or i and j are both less than a.

We'll show that on a weighted global score which assigns equal weight to the at-most propositions (propositions of the form "there are most m of quantity Q") and no others, the at-least propositions (propositions of the form "there are at least m of quantity Q") and no others, or both, and which is derived from a local score that satisfies TRUTH-DIRECTNESS and SYMMETRY, *b* is at least as accurate as *c* at w_a. If condition (ii) holds for some $w_i \in \Omega$, *b* is more accurate than *c* at w_a. This will follow from Result 1, and from the fact that we can think of the magnitude metric as a kind of disagreement metric.

First, note that the distance between any two worlds on the magnitude metric is equal to the distance between any two worlds on a disagreement metric on which the atomic propositions are the at-most propositions (see note 9 in the main text).

Second, note is that if x and y are both greater than a or both less than a (graphically: w_x and w_y are both to the right or both to the left of w_a), then the further of $\{w_x, w_y\}$ to w_a disagrees with w_a about all of the at-most propositions that the closer world disagrees with w_a about. Why? Suppose x and y are both greater than a and, without loss of generality, let $x<y$. Then w_x and w_a disagree with one another about all propositions of the form "there are at most i of quantity Q" when $a \leq i < x$. Similarly, w_y and w_a disagree with one another about all propositions of the form "there are at most i of quantity Q" when $a \leq i < y$. Because $y>x$, for every i such that $a \leq i < x$, it is also true that $a \leq i < y$. Thus, if x and y are both greater than a, every at-most proposition that w_a and w_x disagree about is a proposition that w_a and w_y disagree about. Parallel reasoning shows that the same holds if w_x and w_y are both less than a.

Because the magnitude metric is equivalent to the disagreement metric with the atomic propositions being the at-most propositions, it follows from Result 1 that any global inaccuracy measure that satisfies TRUTH-DIRECTEDNESS and SYMMETRY, and which takes as privileged the at-most propositions satisfies Proximity 2. Similar reasoning applies to measures that take as privileged the at-least propositions, and measures that take both the at-most and the at-least propositions as privileged.

Result 3

When distance between worlds in Ω is given by the disagreement metric, the weighted absolute value score which assigns equal weight to all the atomic propositions and no other propositions satisfies Proximity 3.

Proof of Result 3

Suppose that *b* and *c* are credence functions defined over a finite set of worlds Ω: $\{w_1 \ldots w_n\}$ where distance between worlds is given by the disagreement metric and suppose that *b* and *c* invest equal amounts of credence in w_a (a world in Ω). For distance d, let X_d be the proposition consisting of all and only worlds that are at least d units away from w_a:

$$X_d = \{w \text{ in } \Omega \mid D(w, w_a) \geq d\}.$$

We'll show that on the weighted absolute value score, which assigns equal weight to all the atomic propositions and no weight to any other propositions, the following holds:

If for all propositions X_d, $b(X_d) \leq c(X_d)$, but for some X_d, $b(X_d) < c(X_d)$ then *b* is more accurate than *c* at w_a.

Let the falsehoods concerning the atomic propositions at w_a be $\{P_1 \ldots P_m\}$. (In other words, if the atomic propositions are $\{A_1 \ldots A_m\}$, then if $w_a \in A_i$, $P_i = {\sim}A_i$, and if $w_a \notin A_i$ then $P_i = A_i$). The inaccuracy of *b* at w_a on the weighted absolute value score is just the sum of the inaccuracy of *b* with respect to the P_i:[3]

$$I_{weighted\text{-}abv}(b, w_a) = \sum_{i=1}^{m} b(P_i)$$

Since $b(P_i) = \sum_{w \in P_i} b(w)$

$$I_{weighted\text{-}abv}(b, w_a) = \sum_{i=1}^{m} b(P_i) = \sum_{i=1}^{m} \sum_{w \in P_i} b(w)$$

Now take any world $w \in \Omega$. Let $D(w, w_a) = d_w$

Since w is d_w units away from w_a, w disagrees with w_a about d_w atomic propositions. This means that $b(w)$ will show up d_w times in:

$$\sum_{i=1}^{m} \sum_{w \in P_i} b(w)$$

- once for each P_i that w is a member of.

More generally, then, we can say that

$$I_{weighted\text{-}abv}(b, w_a) = \sum_{w \in \Omega} b(w) D(w, w_a)$$

Now recall that we're assuming that for all X_d (where X_d is the proposition consisting of worlds d or more units away from w_a) $b(X_d) \leq c(X_d)$. So, where Δ is the furthest distance any world is from w_a, we know that

$$\sum_{i=1}^{\Delta} b(X_d) \leq \sum_{i=1}^{\Delta} c(X_d)$$

Now note that if a world is 1-unit away from w_a, it will show up in exactly one X_d proposition (where d ranges between 1 and Δ), namely X_1 – the proposition consisting of worlds 1 or more units away. A world 2 units away from w_a will show up in exactly two such X_d propositions: namely

[3] Note that I'm relying here to the fact that the absolute value score is symmetric.

the proposition consisting of worlds that are at least one unit away (X_1), and the proposition consisting of worlds that are at least two units away (X_2). In general, for any w, if $D(w,w_a) = \delta$, then $b(w)$ will show up in δ of the X_d propositions, with d ranging between 1 and Δ.

So:

$$\sum_{i=1}^{\Delta} b(X_d) = \sum_{w\in\Omega} b(w)D(w, w_a) = I_{weighted\text{-}abv}(b, w_a)$$

For the same reason,

$$\sum_{i=1}^{\Delta} c(X_d) = \sum_{w\in\Omega} c(w)D(w, w_a) = I_{weighted\text{-}abv}(b, w_a)$$

Since:

$$\sum_{i=1}^{\Delta} b(X_d) \leq \sum_{i=1}^{\Delta} c(X_d),$$

it follows that

$$I_{weighted\text{-}abv}(b, w_a) \leq I_{weighted\text{-}abv}(c, w_a)$$

If the inequality is strict, strict inequality follows.

Result 4

When distance between worlds in Ω is given by the disagreement metric, the weighted Brier score which assigns equal weight to all the atomic propositions and no weight to any other propositions satisfies Proximity 4. When distance between worlds in Ω is given by the magnitude metric, the weighted Brier score which assigns equal weight to all the at most propositions and no other propositions, all the at least propositions and no other propositions, or both the at-most and at-least propositions and no other propositions satisfies Proximity 4.

Proof of Result 4

Suppose that b and c are probability distributions over a finite set of worlds Ω, that $w_a \in \Omega$ and that b and c invest equal amount of credence in w_a. Suppose also that b distributes its credence amongst the inaccuracy-determining propositions at w_a at least as evenly as c as defined by Brier-entropy.
For, distance d, let X_d be the proposition (set of worlds) consisting of all and only worlds that are at least d units away from w_a: $X_d = \{w \text{ in } \Omega \mid D(w, w_a) \geq d\}$. We'll show that if for all propositions X_d, $b(X_d) \leq c(X_d)$, but for some X_d, $b(X_d) < c(X_d)$ then b is more accurate than c at w_a.
Let the F_i be the inaccuracy-determining propositions at world w_a.
By Result 3 and its corollary we know that b is more accurate than c on the weighted absolute value score: Thus

$\sum b(F_i) < \sum c(F_i)$

And so

$(\sum b(F_i))^2 < (\sum c(F_i))^2$

If b is at least as evenly distributed as c amongst F_i:

$[\sum b(F_i)^2/(\sum b(F_i))^2] \geq [\sum c(F_i)^2/(\sum c(F_i))^2]$

Since we've established that the denominator on the left is less than the denominator on the right, for b to be at least as evenly distributed as c, the numerator on the right must be greater than the numerator on the left:

$\sum c(F_i)^2 > \sum b(F_i)^2$

But the terms on either side of the inequality are just the inaccuracy scores of c and b respectively according to the weighted Brier score at w_a. So b is more accurate than c at w_a on the weighted Brier score.

Result 5
The global Brier score that assigns equal weight to all convex propositions and no weight to any other propositions satisfies Proximity 5.

Proof of Result 5
Let Ω be a finite space of worlds where distance between worlds is given by the magnitude metric. Let w_a be a world in Ω and let w_{a+d} and w_{a-d} be two worlds that are d units away from w_a. Suppose b and c are credence functions that invest equal amounts of credence in w_a and which are such that b distributes its credence at least as evenly among non-w_a worlds as c does. We'll show that if b invests all of its non-w_a credence in the worlds that are d units away from w_a and c invests all of its non-w_a credence in worlds that are at least d units from w_a, and there is some world in which c invests positive credence that is more than d units away from w_a, b is more accurate than c at w_a.

Let $b(w_i) = b_i$ and $c(w_i) = c_i$

Then Ω looks like this:

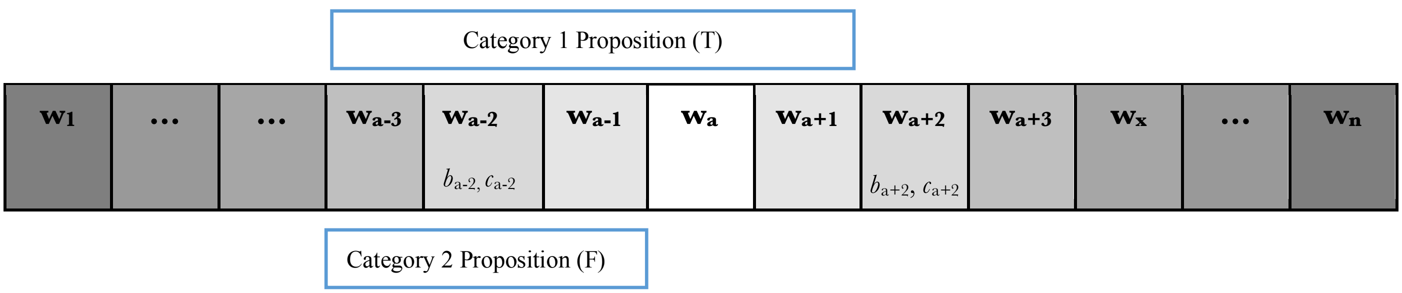
w_1	…	…	…	w_{a-d}	…	w_a	…	w_{a+d}	…	w_x	…	w_n
				b_{a-d}, c_{a-d}				b_{a+d}, c_{a+d}		c_x		

We'll now consider all the convex propositions to which b assigns a non-extreme credence (that is, the convex propositions to which b does not assign 0 or 1). Each such proposition belongs to one of the following four categories:

Category 1: A true proposition to which b assigns $b_{a\text{-}d}$
Category 2: A false proposition to which b assigns $b_{a\text{-}d}$
Category 3: A true proposition to which b assigns b_{a+d}
Category 4: A false proposition to which b assigns b_{a+d}

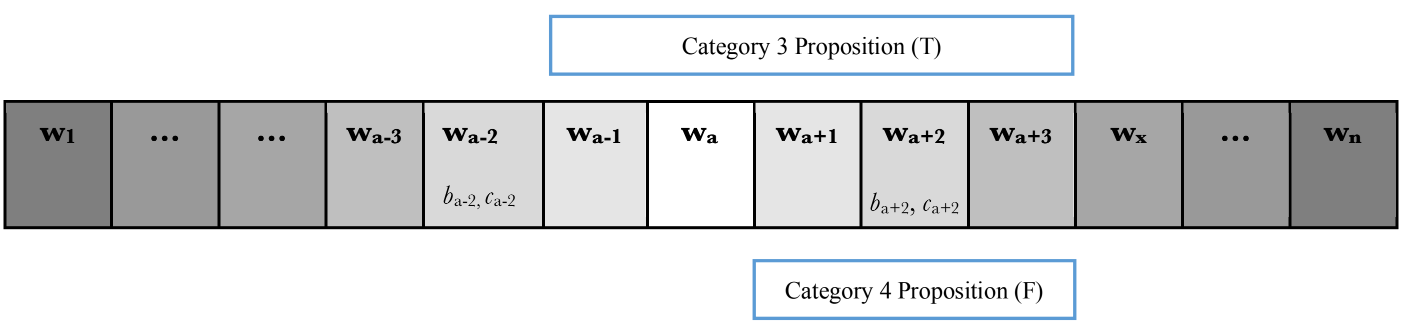
We'll first show that there is a 1-1 mapping between convex propositions in Categories 1 and 2, as well as a 1-1 mapping between convex propositions in Categories 3 and 4.

Take any proposition in Category 1. Such a proposition will be a set of worlds $[w_{a\text{-}d\text{-}j}, w_{a+k}]$ for some $0 \leq j < a\text{-}d$ and for some $0 \leq k < d$. Each such proposition gets mapped to a convex proposition in Category 2. Which one? The proposition with the same left-hand border as the Category 1 proposition, but to turn the proposition from a true one into a false one, the right-hand-border, instead of being k units to the right of w_a, is k units to the left of w_a. For example:



In other words, the Category 1 proposition $[w_{a\text{-}d\text{-}j}, w_{a+k}]$ gets mapped on to the Category 2 proposition: $[w_{a\text{-}d\text{-}j}, w_{a\text{-}k}]$. We'll call these two propositions "partners." The partners of the propositions in Category 1 exhaust the propositions in Category 2.

Now take any proposition in Category 3: A true proposition to which b assigns b_{a+d}. Such a proposition will be a set of worlds $[w_{a\text{-}j}, w_{a+d+k}]$ for some $0 \leq j < d$, and for some $0 \leq k \leq n\text{-}(a+d)$. Each such proposition gets mapped to a convex proposition in Category 4. Which one? One with the same right-hand-border as the Category 3 proposition, but to turn the proposition from true to false, the left-hand-border, instead of being j units to the left of w_a, will be j units to the right of w_a. For example:



In other words, the Category 3 proposition $[w_{a-j}, w_{a+d+k}]$ gets mapped on to the Category 4 proposition: $[w_{a+j}, w_{a+d+k}]$. We'll call these two propositions "partners." The partners of the propositions in Category 3 exhaust the propositions in Category 4.

Now, if we take any Category 1 proposition, P (which, recall, is true), *b*'s inaccuracy with respect to P is b_{a+d}^2. Why? Because the only world which is *not* in the true proposition P that *b* assigns positive credence to is w_{a+d}.

If we take the partner proposition of P, which is the false proposition P', *b*'s inaccuracy with respect to P' is b_{a-d}^2. Why? Because the only world in the false proposition P' that *b* assigns positive credence to is w_{a-d}.

Similarly for Categories 3 and 4. *b*'s inaccuracy with respect to a Category 3 proposition, P, is b_{a-d}^2. And *b*'s inaccuracy with respect to its partner in Category 4 is b_{a+d}^2.

So for all propositions P in Categories 1-4

$$I_{l\text{-}brier}(b(P), w_a(P)) + I_{l\text{-}brier}(b(P'), w_a(P')) = b_{a-d}^2 + b_{a+d}^2$$

Because of all the n worlds in the space, *b* only invests positive credence in w_{a-d} and w_{a+d}

$$b_{a-d}^2 + b_{a+d}^2 = \sum_{i=1}^{n} b_i^2$$

From the above two equations it follows that:

$$I_{l\text{-}brier}(b(P), w_a(P)) + I_{l\text{-}brier}(b(P'), w_a(P')) = \sum_{i=1}^{n} b_i^2$$

Let's now consider *c*'s inaccuracy score with respect to convex propositions.
Take a proposition P in Category 1: $[w_{a-d-j}, w_{a+k}]$ for some $0 \leq j < a-d$, and for some $0 \leq k < d$. Since this proposition is true at w_a, the credences that contribute to *c*'s inaccuracy with respect to this proposition are all the positive credences invested in worlds that are *not* members of this set: worlds in $[w_1, w_{a-d-j-1}]$ as well as in the worlds in $[w_{a+k+1}, w_n]$. However, since we're assuming that $k<d$ and that *c* invests no positive credence in worlds that are fewer than d units away from w_a, it follows that *c* invests no positive credence in $[w_{a+k+1}, w_{a+d-1}]$. Thus the worlds in $[w_{a+k+1}, w_n]$ that contribute to *c*'s inaccuracy are all members of $[w_{a+d}, w_n]$, and so the worlds that contribute to *c*'s inaccuracy in Ω are those in: $[w_1, w_{a-d-j-1}]$ and $[w_{a+d}, w_n]$.

Now consider this proposition's partner P' (in Category 2): $[w_{a-d-j}, w_{a-k}]$. Since this proposition is false, the credences that contribute to *c*'s inaccuracy with respect to that proposition are the positive credences invested in worlds that are members of this set. Since *c* doesn't invest any credence in worlds that are less than d units away from w, it doesn't invest any credence in worlds in $[w_{a-d+1}, w_{a-k}]$. Thus, the credences that contribute to *c*'s inaccuracy with respect to this proposition will be the worlds in $[w_{a-d-j}, w_{a-d}]$. So looking at the inaccuracy of P and P' we have:

$$I_{l\text{-}brier}(c(P), w(P)) = \left(\sum_{i=1}^{a-d-j-1} c_i + \sum_{i=a+d}^{n} c_i\right)^2 > \left(\sum_{i=1}^{a-d-j-1} c_i^2 + \sum_{i=a+d}^{n} c_i^2\right)$$

$$I_{l\text{-}brier}(c(P'), w(P')) = \left(\sum_{i=a-d-j}^{a-d} c_i\right)^2$$

It follows that

$$I_{l\text{-}brier}(c(P), w(P)) + I_{l\text{-}brier}(c(P'), w(P')) > \sum_{i=a}^{a-d-j-1} c_i^2 + \sum_{i=a+d}^{n} c_i^2 + \sum_{i=a-d-j}^{a-d} c_i^2$$

Reordering, (and noting that for all i∈ (a-d, a+d), $c_i = 0$):

$$= \sum_{i=1}^{a-d-j-1} c_i^2 + \sum_{i=a-d-j}^{a-d} c_i^2 + \sum_{a+d}^{n} c_i^2 = \sum_{i=1}^{n} c_i^2$$

Thus,

$$I_{l\text{-}brier}(c(P), w(P)) + I_{l\text{-}brier}(c(P'), w(P')) > \sum_{i=1}^{n} c_i^2$$

So here's where we are: for each proposition P, in our first two categories, the sum of the inaccuracy scores of *b* with respect to P, and with respect to its partner P' is the sum of the b_i^2, whereas the sum of the inaccuracy scores of *c* with respect to P and with respect to its partner P' is greater than the sum of the c_i^2. Since *b* is at least as evenly distributed as *c*, we know that:

$$1- \left[\sum_{i=1}^{n} b(w_i)^2 / \left(\sum_{i=1}^{n} b(w_i)\right)^2\right] \geq 1- \left[\sum_{i=1}^{n} c(w_i)^2 / \left(\sum_{i=1}^{n} c(w_i)\right)^2\right]$$

Since $\sum b(w_i) = \sum c(w_i) = 1$, it follows that $\sum b_i^2 \leq \sum c_i^2$.

Since the inaccuracy of *b* with respect to P and P' = $\sum b_i^2$, and the inaccuracy of *c* with respect to P and P' is *greater* than $\sum c_i^2$, it follows from the fact that $\sum b_i^2 \leq \sum c_i^2$, that *b*'s inaccuracy with respect to these two propositions is greater than *c*'s inaccuracy with respect to these two propositions.

An analogous argument applies to propositions in the Categories 3 and 4. Thus, *b*'s inaccuracy with respect to all the propositions in these 4 categories is less than *c*'s total inaccuracy with respect to all the propositions in these 4 categories.

It remains to consider convex propositions to which *b* assigns credence 1 or 0. There are three categories:

Category 5: True convex propositions to which *b* assigns credence 1

Category 6: True convex propositions to which *b* assigns credence 0
Category 7: False convex propositions to which *b* assigns credence 0.
(Note that there are no false convex propositions to which *b* assigns credence 1).

Let's compare *b* and *c*'s inaccuracy with respect to propositions in each of these three categories.
Every proposition in Category 5 is a true proposition to which *b* assigns credence 1, and so *b* gets inaccuracy score 0 with respect to these propositions. Some of these propositions will be ones such that *c* assigns credence 1 to them as well. But there will be at least one proposition to which *b* assigns credence 1 and which is such that *c* assigns credence less than 1. For example: the proposition [w_{a-d}, w_{a+d}] is one to which *b* assigns credence 1, but *c* assigns credence less than 1 (since *c* invests at least some positive credence in worlds that more than d-units away from w_a).
Every proposition in Category 6 is a true proposition to which *b* assigns credence 0, and so *b* gets inaccuracy score 1 with respect to these propositions. Each such proposition is one that *c* also assigns credence 0 to (since *c* doesn't invest any credence in worlds that are less than d-units away from w_a). Thus, *b* and *c* tie with respect to each proposition in this category.
Finally, let's consider Category 7: false propositions to which *b* assigns credence 0, and so gets inaccuracy score 0. Some of these propositions may be ones to which *c* also assigns 0. But there will be at least one false proposition such that *b* assigns credence 0 to it, and to which *c* assigns positive credence. Consider a world in which *c* assigns positive credence that is more than d-units away from w_a, and call it "w_{a+d+m}." The proposition {w_{a+d+m}} will be a false convex proposition to which *b* assigns credence 0 and *c* assigns positive credence.

Since across each of categories 5 and 7 *b* is less inaccurate than *c* at w_a and across category 6, *b* and *c* are equally accurate, if we consider *b*'s inaccuracy across propositions in categories 5-7, *b* will be less inaccurate than *c* at w_a. We already established that *b* is less inaccurate than *c* across categories 1-4. Since categories 1-7 exhaust all the convex propositions, *b* is less inaccurate than *c* at w_a on the weighted Brier score which assigns equal weight to all the convex propositions and no weight to any other propositions.